November 2, 2004

University of Saskatchewan
Department of Electrical Engineering

EE301 Electricity, Magnetism and Fields
Midterm Examination
Professor Robert E. Johanson

Welcome to the EE301 Midterm. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **2** hours.

Each problem is worth 25 points; if subparts are weighted differently, the points for each are shown in parentheses. Show your work; credit will be given only if the steps leading to the answer are **clearly** shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers. Be reasonably neat; credit will not be given for illegible answers.

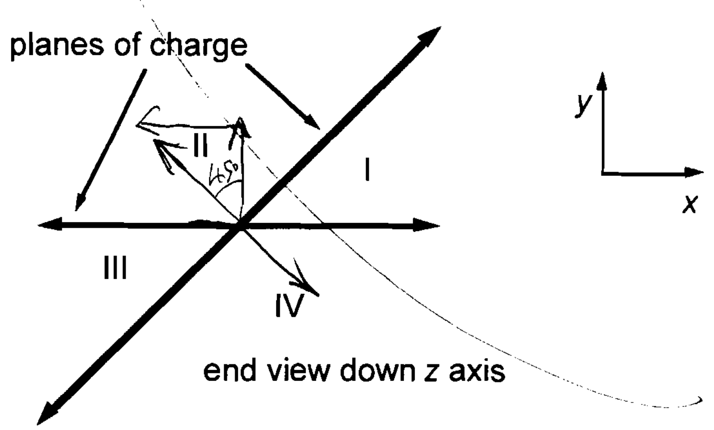
You are to answer **4** of the 6 problems as follows: answer any three of problems 1 to 4 and also either problem 5 or problem 6. Do not solve more than four problems or severe penalties will apply.

None of the problems require intricate mathematical manipulations. Cartesian coordinate triples are always (x, y, z).

Answer any three of problems 1 to 4 and answer either problem 5 or problem 6.

Problem 1

Two infinite planes of charge intersect at 45° as shown. Each plane has a uniform surface charge density of +10 nC/m². Determine the electric field in each of the four regions I, II, III, IV. Use the coordinate system shown in the figure.



Problem 2

A charge of 10 nC is at (1, 0, 0) and charges of −10 nC are at (0, 1, 0) and (0, 0, 1). Determine the electric field vector and the electric potential at the point (1, 1, 1). (Cartesian coordinates, distances in meters)

Problem 3

a) (8 pts) A charge distribution expressed in cylindrical coordinates depends only on the variable ρ. What does symmetry imply about the electric field $\vec{E}(\rho,\varphi,z)$ produced by this charge?

b) (17 pts) Within a cylindrical region, the charge density is given by

$$\rho_V = \frac{2Q}{\pi a^2}\left(1-\frac{\rho^2}{a^2}\right) \text{ for } \rho < a.$$

$\rho_V = 0$ outside this region. Use Gauss's Law to calculate the electric field both inside and outside the charge.

didn't do

Problem 4

Two concentric, spherical shells of metal form a capacitor. The inner shell has radius r_1 and the outer shell has radius r_2. A dielectric with dielectric constant ε_R fills the space between the two shells.

a) (9 pts) The inner shell has charge Q and the outer shell has charge $-Q$. What is the surface charge density on each shell? What is the electric field inside the inner shell, between the two shells, and outside the shells?

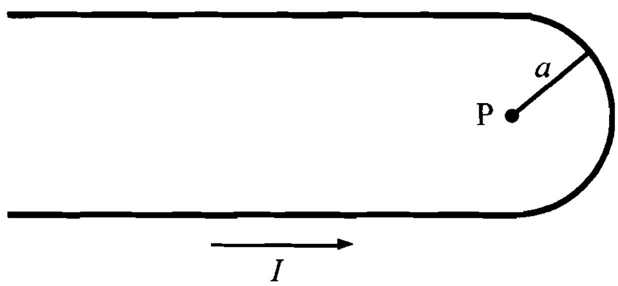
b) (16 pts) Determine the formula for the capacitance. How does the capacitance of the spherical capacitor relate to that of the parallel plate capacitor when the separation between the shells is small compared to the radius, $r_2 - r_1 << r_1$?

Problem 5

The middle of an infinitely long wire is bent in a half circle as shown; the radius of the bend is a. A current I flows in the wire. Calculate the magnetic field vector at the center of the bend P.

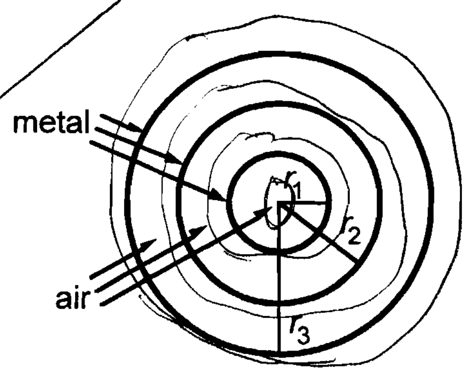
You might be interested to know that $\int_0^\infty \frac{dx}{(a^2+x^2)^{3/2}} = \frac{1}{a^2}$ although if you are clever you won't need it.

Didn't do



Problem 6

A triaxial cable consists of three concentric, hollow, cylindrical conductors of radii r_1, r_2, and r_3. A current of $I/2$ flows through the inner and outer conductor and a current of $-I$ (i.e. in the opposite direction) flows through the center conductor. Use Ampere's Law to calculate the magnetic field everywhere.



capacitance	$C = Q/V$
parallel plate cap.	$C = \dfrac{\varepsilon_0 \varepsilon_R A}{d}$ of area A and separation d
Poisson's equation	$\nabla^2 V = -\rho / \varepsilon_0 \varepsilon_R$
Laplace's equation	$\nabla^2 V = 0$
linear dielectrics	$\vec{D} = \varepsilon_0 \varepsilon_R \vec{E}$
dielectric b.c.	E_T and D_N continuous across boundary
energy	$W = \dfrac{1}{2} \int_V \vec{E} \cdot \vec{D} \, dV$

Magnetostatics

law of Biot-Savart	$\vec{H} = \oint \dfrac{I \, d\vec{l} \times \vec{a}_R}{4\pi R^2}$
	$\vec{H} = \int_V \dfrac{\vec{j} \times \vec{a}_R \, dV}{4\pi R^2}$
Ampere's law	$\oint \vec{H} \cdot d\vec{l} = I_{\text{enclosed}} = \int_S \vec{j} \cdot d\vec{S}$
inductance	$L = N\phi / I$
vector potential	$\vec{A} = \int_V \dfrac{\mu_0 \vec{j} \, dV}{4\pi R}$
relating $\vec{B}$ to $\vec{A}$	$\vec{B} = \vec{\nabla} \times \vec{A}$
linear materials	$\vec{B} = \mu_0 \mu_R \vec{H}$
b.c.	H_T and B_N continuous across boundary
energy	$W = \dfrac{1}{2} \int_V \vec{B} \cdot \vec{H} \, dV$

Electromagnetics

Maxwell's equations

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

Symbols and Constants

F	force	V	electric potential
Q	charge	$\vec{A}$	vector potential
$\vec{E}$	electric field	ρ_V	volume charge density
$\vec{D}$	displacement field	I	current
$\vec{P}$	polarization field	$\vec{j}$	current density
$\vec{H}$	magnetic field	ε_R	relative permittivity
$\vec{B}$	magnetic flux density field	μ_R	relative permeability
$\vec{M}$	magnetization field		$\varepsilon_0 \approx 8.85 \times 10^{-12}$ F/m
			$\mu_0 = 4\pi \times 10^{-7}$ N/A^2

Vector Calculus

cross products

Cartesian $\quad \vec{a}_x \times \vec{a}_y = \vec{a}_z \quad \vec{a}_y \times \vec{a}_z = \vec{a}_x \quad \vec{a}_z \times \vec{a}_x = \vec{a}_y$

cylindrical $\quad \vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z \quad \vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho \quad \vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi$

spherical $\quad \vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi \quad \vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r \quad \vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta$

spheres

$$Vol = (4/3)\pi r^3 \quad Area = 4\pi r^2$$

$$dV = r^2 \sin\theta\, dr d\theta d\varphi \quad dS = r^2 \sin\theta\, d\theta d\varphi$$

cylinders $\quad dV = \rho\, d\rho d\varphi dz$

Electrostatics

Coulomb's law

$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\vec{a}_{12}$$

point charge

$$\text{field: } \vec{E} = \frac{Q\vec{a}_r}{4\pi\varepsilon_0 r^2} \qquad \text{potential: } V = \frac{Q}{4\pi\varepsilon_0 r}$$

charge distribution

$$\vec{E} = \int_V \frac{\rho_V(\vec{r}\,')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|^2} \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|} dV' \qquad V = \int_V \frac{\rho_V(\vec{r}\,')\, dV'}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|}$$

Gauss's law

$$\varepsilon_0 \oint_S \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{total}} = \int_V \rho_V\, dV$$

$$\oint_S \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{free}} = \int_V \rho_V^{\text{free}}\, dV$$

$$\oint_S \vec{P} \cdot d\vec{S} = -Q_{\text{enclosed}}^{\text{bound}} = -\int_V \rho_V^{\text{bound}}\, dV$$

relating $\vec{E}$ and V

$$\vec{E} = -\vec{\nabla} V \qquad V_{AB} = -\int_B^A \vec{E} \cdot d\vec{l}$$

UNIVERSITY OF SASKATCHEWAN
EXAMINATION BOOKLET

PLEASE PRINT CLEARLY

SURNAME NESS

FIRSTNAME KYLE

STUDENT NUMBER 9 6 1 0 1 6

CLASS (e.g. BIOL 110 02)

SUBJECT	COURSE NUMBER	SECTION NUMBER
E E	3 0 1	0 1

BOOKLET NUMBER 1

TOTAL NUMBER OF BOOKLETS HANDED IN 1

I declare that I am the person named. I am formally registered as a student in the class indicated on this cover page.

SIGNATURE Kyle Ness

DATE November 3, 2004

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1.	25
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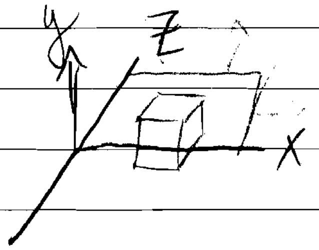
TOTAL	100%

Revised November 2000

Problem #1

First consider a plane by itself then use superposition.



Using Gauss's Law

$E(\cancel{x},y,\cancel{z}) = \cancel{E_x \vec{a_x}} + E_y \vec{a_y} + \cancel{E_z \vec{a_z}}$

$E(y) = E_y \vec{a_y}$ By Symmetry

25

$dl = dx\,dz\,\vec{a_y}$

$\epsilon_0 \int_{Top} E_y\,dx\,dz\,\vec{a_y}\cdot\vec{a_y}^{\,1} + \epsilon_0 \int_{bottom} E_y\,dx\,dz\,a_y\cdot\vec{a_y}^{\,1}$

$= 2\epsilon_0 E_y a$ where a is the area of the top side and also the area of the bottom side of the box.

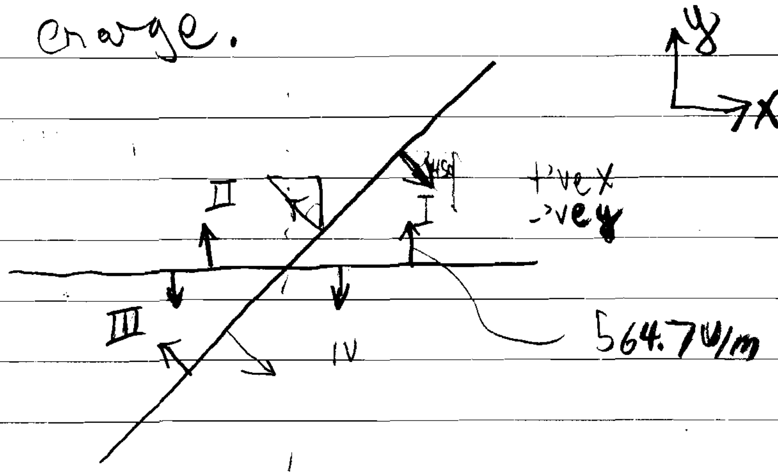
$Q_{enclosed} = \frac{10nc}{m^2}\,a$

$\therefore E_y = \frac{10nc\,a}{2\epsilon_0 a}\,\vec{a_y}$ for $y > 0$

$E_y = \frac{-10nC}{2\epsilon_0}\,\vec{a_y}$ for $y < 0$

$= 564.7\ \frac{V}{m}$

This field will be the same for the plane at a 45° angle only it will be in the x and y directions due to the planes 45° orientation and the fact that the $\vec{E}$ is $\perp$ to an infinite plane of charge.



I

$$\cos(45°)\ \frac{10nC}{2\epsilon_0}\ \vec{a_x} + (\sin 45°)\ \frac{10nC}{2\epsilon_0}\ \vec{a_y} + \frac{10nC}{2\epsilon_0}\ \vec{a_y}$$

$$\boxed{\vec{E} = 399.3\,\vec{a_x} + 165.4\,\vec{a_y}\ V/m}$$ ✓

II

$$\boxed{\vec{E} = -399.3\,\vec{a_x} + 964\,\vec{a_y}\ V/m}$$ ✓

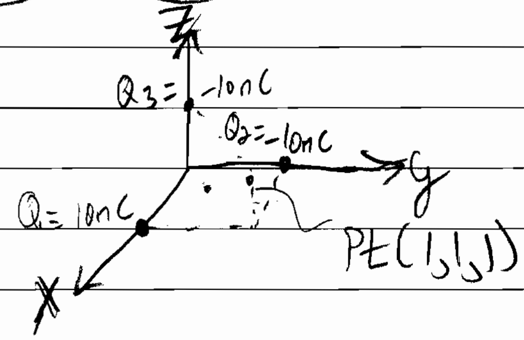
III

$$\boxed{\vec{E} = -399.3\,\vec{a_x} - 165.4\,\vec{a_y}\ V/m}$$ ✓

IV

$$\boxed{\vec{E} = 399.3\,\vec{a_x} - 964\,\vec{a_y}\ V/m}$$ ✓

Problem #2



$\vec{r} = \vec{a_x} + \vec{a_y} + \vec{a_z}$

$\vec{r}_1' = \vec{a_x}$

$r_2' = \vec{a_y}$

$r_3' = \vec{a_z}$

Using Superposition

Charge 10nC (1,0,0)

$\vec{r} - r_1' = \vec{a_y} + \vec{a_z}$

$|\vec{r} - \vec{r}_1'| = \sqrt{2}$

$\vec{E}_1 = \dfrac{Q_1}{4\pi\epsilon_0(\sqrt{2})^2}\left(\dfrac{\vec{a_y} + \vec{a_z}}{\sqrt{2}}\right)$

Charge Q_2

$\vec{r} - \vec{r}_2' = \vec{a_x} + \vec{a_z}$

$|\vec{r} - \vec{r}_2'| = \sqrt{2}$

$\vec{E}_2 = \dfrac{Q_2}{4\pi\epsilon_0(\sqrt{2})^2}\left(\dfrac{\vec{a_x} + \vec{a_z}}{\sqrt{2}}\right)$

Charge Q_3

$\vec{r} - \vec{r}_3' = \vec{a_x} + \vec{a_y}$

$|\vec{r} - \vec{r}_3'| = \sqrt{2}$

$\vec{E}_3 = \dfrac{Q_3}{4\pi\epsilon_0(\sqrt{2})^2}\left(\dfrac{\vec{a_x} + \vec{a_y}}{\sqrt{2}}\right)$

By looking at the picture you can see that the forces in y and z directions cancel

$E_{tot} = E_1 + E_2 + E_3$

$$\vec{E} = \frac{1}{8\pi\epsilon_0\sqrt{2}}\left[(Q_2 + Q_3)\vec{a_x} + \underbrace{(Q_1 + Q_3)}_{0}\vec{a_y} + \underbrace{(Q_1 + Q_2)}_{0}\vec{a_z}\right]$$

$$\boxed{\vec{E} = -63.55\,\vec{a_x}\ \frac{V}{m}}$$

✓

Potential use superposition

$V_1 = \frac{10nC}{4\pi\epsilon_0\sqrt{2}}$ $V_2 = \frac{-10nC}{4\pi\epsilon_0\sqrt{2}}$ $V_3 = \frac{-10nC}{4\pi\epsilon_0\sqrt{2}}$

$V_T = V_1+V_2+V_3 = \frac{-10nC}{4\pi\epsilon_0\sqrt{2}} = \boxed{-63.55\ V}$ ✓

We can also find this by looking at the $\vec{E}$ and seeing that ~~y~~ and z coordinates don't matter for electric field. We then see that our point is 1m away from the origin in the +'ve x direction and so should have a potential of $-63.55\frac{V}{m}(1m) = -63.55V$.

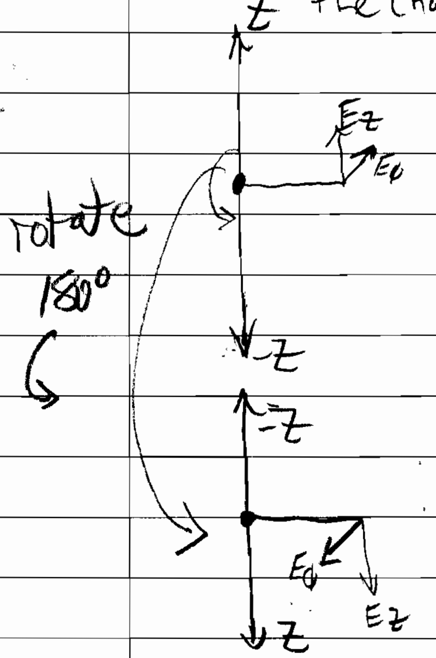
not entirely clear

problem #3

a) $\vec{E}(\rho,\cancel{\phi},\cancel{z}) = E_\rho \vec{a_\rho} + \cancel{E_\phi \vec{a_\phi}} + \cancel{E_z \vec{a_z}}$ ✓

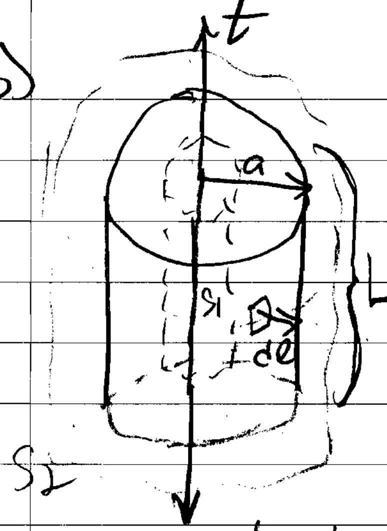
If our charge distribution only depends on ρ that means it must look like an infinitely long cylinder or line of charge in z direction. This is b/c then the z component won't matter b/c the charge looks the same no matter what z you are at since it is infinetly long. Also rotating in ϕ doesn't ~~change~~ change charge distribution either since at any angle (same z) the charge looks the same.

25



This also means we have no E_ϕ or E_z since if we flipped our charge 180° about the z=0 axis we would get the same charge distribution but a component in the ϕ direction or z direction would be pointing the opposite direction yielding multiple solutions for a single charge distribution which cannot be. ✓

b)



Assume region of length L

$\vec{E}(\rho,\phi,z) = E_\rho \vec{a_\rho} + \cancel{E_\phi \vec{a_\phi}} + \cancel{E_z \vec{a_z}}$

$\vec{E}(\rho) = E_\rho \vec{a_\rho}$

$d\ell = \rho \, d\phi \, dz \, \vec{a_\rho}$

$\epsilon_0 \int_0^L \int_0^{2\pi} E_\rho \vec{a_\rho} \cdot \rho \, d\phi \, dz \, \vec{a_\rho} = \epsilon_0 \rho E_\rho 2\pi L$

inside charge

Use surface S_1 a cylinder of radius ρ

$$Q_{enclosed} = \int_0^\rho \int_0^{2\pi} \int_0^L \frac{2Q}{\pi a^2}\left(1 - \frac{\rho^2}{a^2}\right) \rho \, dz \, d\theta \, d\rho$$

$$= \int_0^\rho \frac{2\pi L 2Q\rho}{\pi a^2}\left(1 - \frac{\rho^2}{a^2}\right) d\rho = \frac{4LQ}{a^2} \int_0^\rho \rho - \frac{\rho^3}{a^2} \, d\rho$$

$$= \frac{4LQ}{a^2}\left(\frac{1}{2}\rho^2 - \frac{\rho^4}{4a^2}\right)$$

$$E_\rho = \frac{4LQ\rho^2}{a^2 \rho \epsilon_0 2\pi L 2}\left(1 - \frac{\rho^2}{2a^2}\right) \vec{a_\rho} = \boxed{\frac{Q}{a^2 \pi \epsilon_0}\left(\rho - \frac{\rho^3}{2a^2}\right) \vec{a_\rho}}$$

very minor error

Outside charge

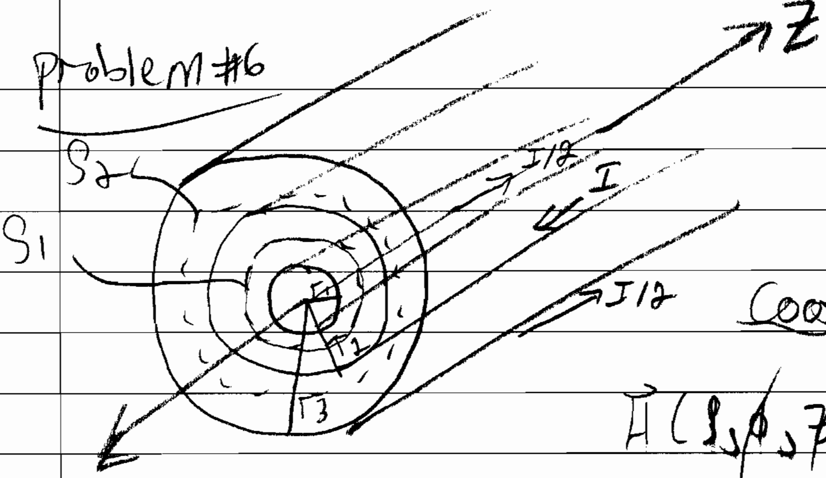
Use surface S_2 a cylinder

$$Q_{enclosed} = \int_0^a \int_0^{2\pi} \int_0^L \frac{2Q}{\pi a^2}\left(1 - \frac{\rho^2}{a^2}\right) \rho \, dz \, d\theta \, d\rho$$

$$= \frac{4LQ}{a^2}\left(\frac{1}{2}a^2 - \frac{a^4}{4a^2}\right) = \frac{4^2 L Q}{2}\left[\frac{a^2}{a^2} - \frac{a^4}{2a^4}\right] = LQ$$

$$E_\rho = \frac{LQ}{\epsilon_0 \rho 2\pi L} = \boxed{\frac{Q}{2\pi \epsilon_0 \rho} \vec{a_\rho}}$$ ✓

problem #6



Using cylindrical Coordinates

$$\vec H(\rho,\phi,z) = H_\rho \cancel{\vec a_\rho} + H_\phi \vec a_\phi + H_z \cancel{\vec a_z}$$

By symmetry

Ampere's Law

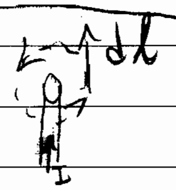
5

$$\oint H \cdot \vec{dl} = I_{enclosed} = \int_s \vec j \cdot \vec{ds}$$

4 regions

$\rho < r_1$ we have no current contained in our loop so $\vec H = 0$.

$\rho > r_3$ we have $I/2 + I/2 - I = 0$ so once again no current contained in our loop so $\vec H = 0$



$$dl = \rho \, d\phi \, \vec a_\phi$$

$$\int_0^{2\pi} H_\phi \rho \, d\phi \, \cancel{\vec a_\phi \cdot \vec a_\phi}^{\,1} = 2\pi \rho H_\phi$$

for $r_1 < \rho < r_2$

$$I_{enclosed} = \frac{I}{2}$$

$$H_\phi = \frac{I/2}{2\pi\rho} = \boxed{\frac{I}{4\pi\rho}\vec a_\phi}$$

for $r_2 < \rho < r_3$

$$I_{enclosed} = \frac{I}{2} - I = -I/2$$

$$H_\phi = \frac{-I/2}{2\pi\rho} = \boxed{\frac{-I}{4\pi\rho}\vec a_\phi}$$